

# **HORIZONTAL EMITTANCE REDUCTION USING A ROBINSON WIGGLER**

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❖ Horizontal (natural) Emittance: 
$$\mathcal{E}_x = \frac{C_q \gamma^2 \langle H \rangle_{dipole}}{J_x \rho_x}$$

$J_x$  = Horizontal damping partition number

❖ Horizontal emittance is determined by the equilibrium between the quantum excitation due to the emission of photons and the damping due to the RF acceleration field used to compensate the energy loss of the synchrotron radiation.

❖  $J_x$  is related to the **damping partition  $D$**  by :  $J_x = 1 - D$

where 
$$D = \frac{\frac{1}{2\pi} \oint \frac{\eta_x(s)}{\rho_x} \left[ \frac{1}{\rho_x^2} + 2K(s) \right] ds}{\oint \frac{ds}{\rho_x^2}}$$

(the integral is to be evaluated only in dipoles).



- ❖ For an isomagnetic storage ring with separate function magnets, where  $K(s) = 0$  in dipoles,

$$D = \frac{1}{2\pi\rho} \oint \frac{\eta(s)}{\rho_x} ds = \frac{\alpha R}{\rho_x}$$

- ❖ Since the momentum compaction  $\alpha \ll 1$  then  $D \ll 1$  for separate function machines.
- ❖ And since  $J_x = 1 - D$
- ❖ Then, for an isomagnetic storage ring with separate function magnets,  $J_x = 1$

Such as :  $J_x + J_z + J_s = 4$  where  $J_z = 1$  and  $J_s = 2$

❖ Looking at the general expression of the damping partition  $D$ ,  $J_x$  and  $\mathcal{E}_x$ :

$$D = \frac{1}{2\pi} \oint \frac{\eta_x(s)}{\rho_x} \left[ \frac{1}{\rho_x^2} + 2K(s) \right] ds$$

$$J_x = 1 - D$$

$$\mathcal{E}_x = \frac{C_q \gamma^2 \langle H \rangle_{dipole}}{J_x \rho_x}$$

❖ If we can make  $D = -1$   $\longrightarrow$   $J_x = 2$

And the horizontal emittance can be divided by a factor 2!

Such as:  $J_x + J_z + J_s = 4$  where  $J_z = 1$  and  $J_s = 1$

- ❖ Note that  $D$  depends on the dispersion function  $\eta(s)$  and the quadrupole strength  $K(s)$

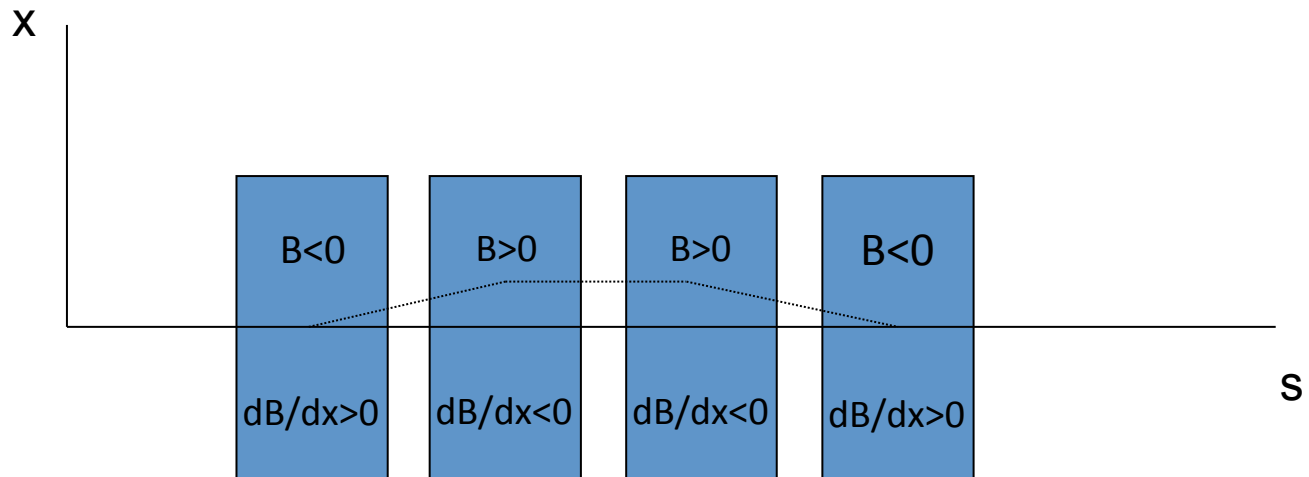
$$D = \frac{\frac{1}{2\pi} \oint \frac{\eta_x(s)}{\rho_x} \left[ \frac{1}{\rho_x^2} + 2K(s) \right] ds}{\oint \frac{ds}{\rho_x^2}}$$

- ❖ And can be re-written:

$$D = \frac{\frac{1}{2\pi} \left( \oint \frac{\eta_x}{\rho_x^3} ds + \frac{2}{B^2 \rho_x^2} \oint \eta_x B \frac{dB}{dx} ds \right)}{\oint \frac{ds}{\rho_x^2}}$$

A magnetic element introducing the product  $B * dB/dx$  in a straight section where the dispersion  $\eta_x$  is non zero contributes to the modification of  $D$ .

- ❖ If this magnetic element is such as the field ( $B$ ) and the gradient ( $dB/dx$ ) are of opposite sign, this contribution is **negative** and consequently we can try to find the conditions to make  $D = -1$ !
- ❖ A **gradient wiggler magnet**, as proposed by Robinson\*, consists of even number of consecutive blocks with **field and gradient of opposite signs**.



\*: K.W. Robinson, Phys. Rev. 3 (1958) 373.

## Radiation Effects in Circular Electron Accelerators\*

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The effects of the radiation emission on the motion of electrons in high-energy synchrotrons are analyzed. The damping rates and quantum excitation of the three principal modes of oscillation are derived for strong focusing and constant gradient accelerators. Methods for correcting the radiation effects for strong-focusing accelerators are discussed.

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### EMITTANCE CONTROL OF THE PS $e^{\pm}$ BEAMS USING A ROBINSON WIGGLER

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In 1958 Robinson of the Cambridge electron accelerator (Massachusetts) proposed that a gradient wiggler magnet be used to stabilise naturally unstable electron and positron beams in combined function machines. In 1986 such a method is to be applied in the PS so that, besides its many other tasks, it may serve as an accelerator in the LEP injector chain.

This paper describes a prototype of a gradient wiggler magnet designed and constructed at CERN. It reports the results of measurements obtained with proton beams in the PS to check the influence of the wiggler on beam optics and of measurements made with positron beams in DCI (LAL, Orsay, France) to check the damping variations produced by this wiggler.

As predictions were confirmed by these measurements, three magnets of this type will be installed in the PS when it is part of the LEP injector chain.

- ❖ Such a wiggler has been used successfully on the PS at CERN (in 1983) to counteract the natural horizontal anti-damping →  $J_x=3$  and  $D = -2$ .

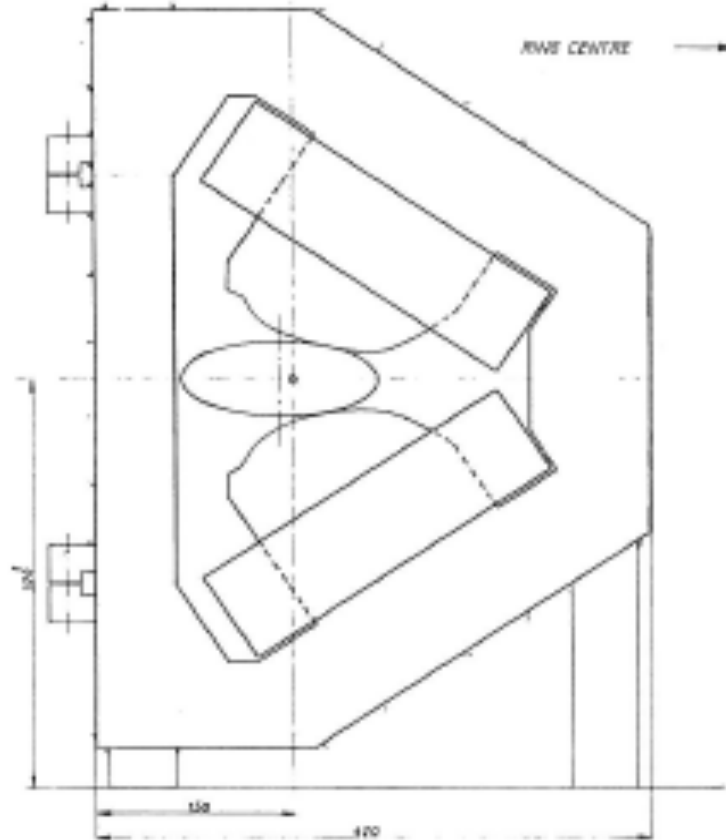
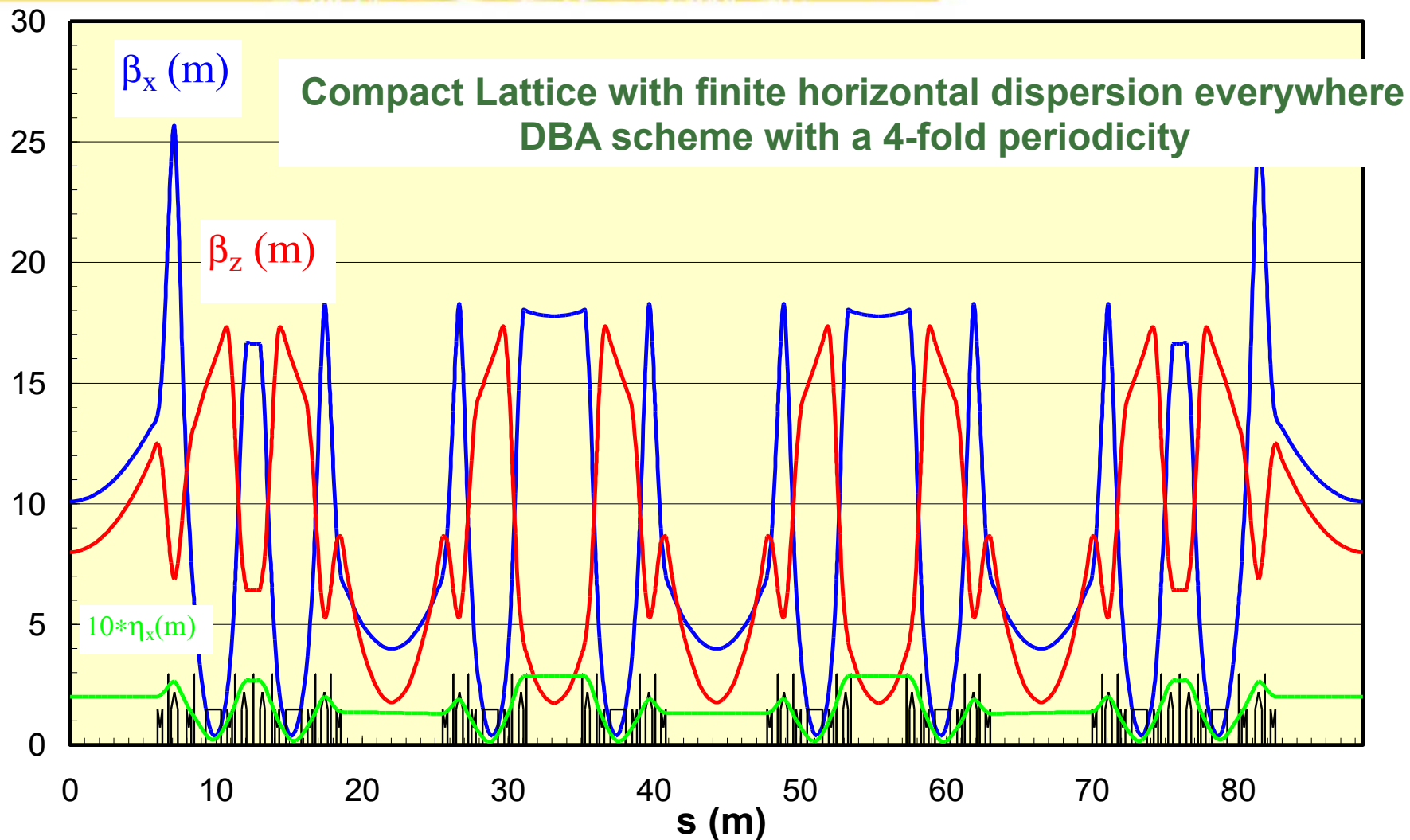


Fig. 10 Cross-section of Robinson wiggler for the PS







- ❖ After some calculations and simplifications one can write the contribution of the wiggler to the damping partition as follows:

$$D = \frac{\rho_0 \eta_x}{\pi(\rho_0 B_0)^2} \left\langle B_w \frac{dB_{w,z}}{dx} \right\rangle L_w$$

$B_w$  = peak field of the wiggler and  $L_w$  its length.

To obtain the maximum effect of damping, the wiggler must be located at a location where the horizontal dispersion has a high value!!!!

- ❖ In short straight sections of the SOLEIL Storage Ring :  $\eta_x = 28 \text{ cm}$

$$D \approx 5.6 \times 10^{-3} \left\langle B_w \frac{dB_{w,z}}{dx} \right\rangle L_w$$

( $B_0 \rho_0 = 9.138 \text{ T.m}$  for SOLEIL)

- ❖ The maximum length of insertion device possible in a short straight section of SOLEIL is  $L_w = 2$  m,
- ❖ By imposing  $D = -1$ , we can study the following examples:

Type	B(T)	g (mm)	dB/dx (T/m)
Out-vacuum	1.4	11	140
In-vacuum	1.0	5.5	182

The magnetic design has just started!

❖ (Normal) wigglers when used in free dispersion sections can reduce the natural emittance of the electron beam. They enhance the radiation damping while making a little contribution to the quantum excitation. **They are called Damping wigglers.**

❖ PETRA-III:

- 6 GeV, 1 nm.rad, large circumference with damping wigglers

❖ NSLS-II:

- 3 GeV, 0.5 nm.rad, large circumference DBA with damping wigglers

❖ MAX-IV:

- 3 GeV, 0.24 nm.rad, 7DBA with damping wigglers

# Drawback and Challenges

❖ Energy dispersion is increased by a factor  $\sqrt{2}$

$$\left(\frac{\sigma_E}{E_0}\right)^2 = \frac{2}{2+D} \left(\frac{\sigma_{E,0}}{E_0}\right)^2$$

❖ Damping in longitudinal plane with  $J_s = 1$  (larger damping time constant)

❖ How to reach such very high gradient?

❖ Homogeneity of the magnetic field in the electron beam region:  
effect on injection and beam lifetime

❖ What are the radiation properties?

- ❖ The use of a **Robinson Wiggler** would be a possibility to **reduce** the horizontal emittance by a **factor 2** in lattices with **finite horizontal dispersion** in straight sections!  
But with an increase on the energy dispersion by a factor  $\sqrt{2}$
- ❖ In principle one is enough!
- ❖ But there are some challenges!
- ❖ At SOLEIL, this study has started with a PhD student.